The Best Theory of Cosmic Structure Formation
is Cold + Hot Dark Matter (CHDM) *

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1   Summary

The fact that the simplest modern cosmological theory, standard Cold Dark Matter (sCDM), almost fits all available data has encouraged the search for variants of CDM that can do better. I have been asked to make the case here that CHDM is the best theory of cosmic structure formation, and indeed I believe that it *is* the best of all those I have considered *if* the cosmological matter density is near critical (i.e., $\Omega_0 \approx 1$) and if the expansion rate is not too large (i.e. $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \lesssim 0.6$). But I think it will be helpful to discuss CHDM together with its chief competitor among CDM variants, low-Ω_0 CDM with a cosmological constant (ΛCDM). While the predictions of COBE-normalized CHDM and ΛCDM both agree reasonably well with the available data on scales of ~ 10 to $100 \ h^{-1}$ Mpc, each has potential virtues and defects. ΛCDM with $\Omega_0 \sim 0.3$ has the possible virtue of allowing a higher expansion rate H_0 for a given cosmic age t_0, but the defect of predicting too much fluctuation power on small scales. CHDM has less power on small scales, so its predictions appear to be in good agreement with data on the galaxy distribution, although it remains to be seen whether it predicts early enough galaxy formation to be compatible with the latest high-redshift data. Also, several sorts of data suggest that neutrinos have nonzero mass, and the variant of CHDM favored by this data — in which the neutrino mass is shared between two species of neutrinos — also seems more compatible with the large-scale structure data. Except for the $H_0 - t_0$ problem, there is not a shred of evidence in favor of a nonzero cosmological constant, only increasingly stringent upper bounds on it from several sorts of measurements. Two recent observational results particularly favor high cosmic

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density, and thus favor $\Omega = 1$ models such as CHDM over ΛCDM — (1) the positive deceleration parameter $q_0 > 0$ measured using high-redshift Type Ia supernovae, and (2) the low primordial deuterium/hydrogen ratio measured in two different quasar absorption spectra. If confirmed, (1) means that the cosmological constant probably cannot be large enough to help significantly with the $H_0 - t_0$ problem; while (2) suggests that the baryonic cosmological density is at the upper end of the range allowed by Big Bang Nucleosynthesis, perhaps high enough to convert the "cluster baryon crisis" for $\Omega = 1$ models into a crisis for low-Ω_0 models. I also briefly compare CHDM to other CDM variants such as Warm Dark Matter (WDM) and tilted CDM. CHDM has the advantage among $\Omega = 1$ CDM-type models of requiring little or no tilt, which appears to be an advantage in fitting recent small-angle cosmic microwave background anisotropy data. The presence of a hot component that clusters less than cold dark matter lowers the effective Ω_0 that would be measured on small scales, which appears to be in accord with observations, and it may also avoid the discrepancy between the high central density of dark matter halos from CDM simulations compared to evidence from rotation curves of dwarf spiral galaxies.

2   Introduction

"Standard" $\Omega = 1$ Cold Dark Matter (sCDM) with $h \approx 0.5$ and a near-Zel'dovich spectrum of primordial fluctuations [1] until a few years ago seemed to many theorists to be the most attractive of all modern cosmological models. But although sCDM normalized to COBE nicely fits the amplitude of the large-scale flows of galaxies measured with galaxy peculiar velocity data [2], it does not fit the data on smaller scales: it predicts far too many clusters [3] and does not account for their large-scale correlations [4], and the shape of the power spectrum $P(k)$ is wrong [5, 6]. Here I discuss what are perhaps the two most popular variants of sCDM that might agree with all the data: CHDM and ΛCDM. The linear *matter* power spectra for these two models are compared in Figure 1 (from Ref. [7]) with the real-space *galaxy* power spectrum obtained from the two-dimensional APM galaxy power spectrum [5]. The ΛCDM and CHDM models essentially bracket the range of power spectra in currently popular cosmological models which are variants of CDM.

CHDM cosmological models have $\Omega = 1$ mostly in cold dark matter but with a small admixture of hot dark matter, light neutrinos contributing $\Omega_\nu = m_{\nu,tot}/(92h^2\text{eV}) \approx 0.2$, corresponding to a total neutrino mass of $m_{\nu,tot} \approx 5$ eV for $h = 0.5$. CHDM models are a good fit to much observational data [8, 9] — for example, correlations of galaxies and clusters and direct measurements of the power spectrum $P(k)$, velocities on small and large scales, and other statistics such as the Void Probability Function (probability $P_0(r)$ of finding no bright galaxy in a randomly placed sphere of radius r). My colleagues and I had earlier shown that CHDM with $\Omega_\nu = 0.3$ predicts a VPF larger than
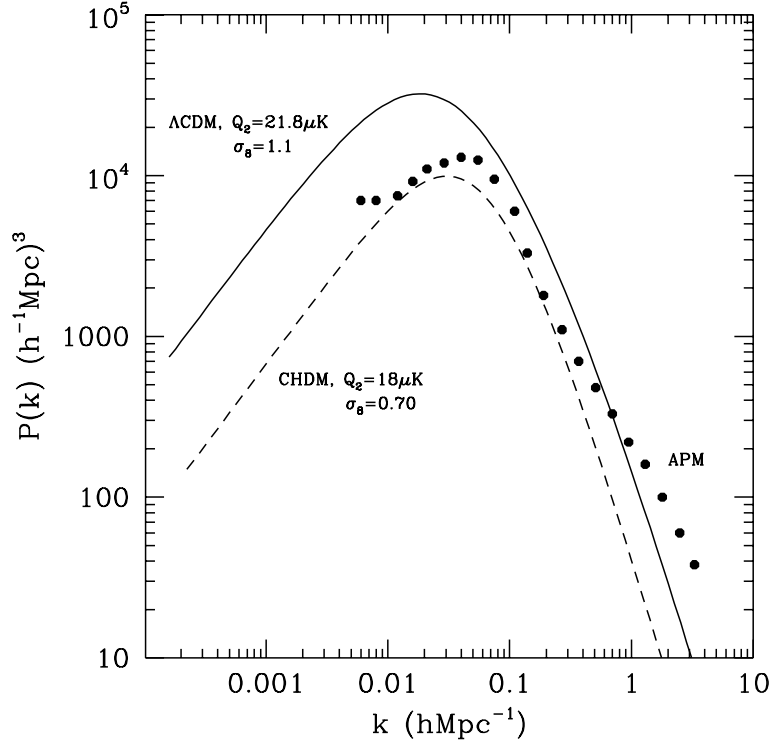
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Figure 1: Power spectrum of dark matter for ΛCDM and CHDM models considered in this paper, both normalized to COBE, compared to the APM galaxy real-space power spectrum. (ΛCDM: $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, thus $t_0 = 13.4$ Gy; CHDM: $\Omega = 1$, $\Omega_\nu = 0.2$ in $N_\nu = 2$ ν species, $h = 0.5$, thus $t_0 = 13$ Gy; both models fit cluster abundance with no tilt, i.e. $n_p = 1$. From Ref. [7].)

observations indicate [10], but new results based on our $\Omega_\nu = 0.2$ simulations in which the neutrino mass is shared equally between two neutrino species [8] show that the VPF for this model is in excellent agreement with observations. However, our simulations [12] of COBE-normalized ΛCDM with $h = 0.7$ and $\Omega_0 = 0.3$ lead to a VPF which is too large to be compatible with the data [11].

Moreover, there is mounting astrophysical and laboratory data suggesting that neutrinos have non-zero mass [8, 13]. The analysis of the LSND data through 1995 [14] strengthens the earlier LSND signal for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations. Comparison with exclusion plots from other experiments implies a lower limit $\Delta m_{\mu e}^2 \equiv |m(\nu_\mu)^2 - m(\nu_e)^2| \gtrsim 0.2$ eV^2, implying in turn a lower limit $m_\nu \gtrsim 0.45$ eV, or $\Omega_\nu \gtrsim 0.02(0.5/h)^2$. This implies that the contribution of hot dark matter to the cosmological density is larger than that of all the visible stars ($\Omega_* \approx 0.004$ [15]). More data and analysis are needed from LSND's $\nu_\mu \to \nu_e$ channel before the initial hint [16] that $\Delta m_{\mu e}^2 \approx 6$ eV^2 can be confirmed. Fortunately the KARMEN experiment has just added shielding to decrease its

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background so that it can probe the same region of $\Delta m^2_{\mu e}$ and mixing angle, with sensitivity as great as LSND's within about two years. The Kamiokande data [17] showing that the deficit of $E > 1.3$ GeV atmospheric muon neutrinos increases with zenith angle suggests that $\nu_\mu \to \nu_\tau$ oscillations [18] occur with an oscillation length comparable to the height of the atmosphere, implying that $\Delta m^2_{\tau\mu} \sim 10^{-2}$ eV^2 [17] — which in turn implies that if either ν_μ or ν_τ have large enough mass ($\gtrsim 1$ eV) to be a hot dark matter particle, then they must be nearly degenerate in mass, i.e. the hot dark matter mass is shared between these two neutrino species. The much larger Super-Kamiokande detector is now operating, and we should know by about the end of 1996 whether the Kamiokande atmospheric neutrino data that suggested $\nu_\mu \to \nu_\tau$ oscillations will be confirmed and extended [19]. Starting in 1997 there will be a long-baseline neutrino oscillation disappearance experiment to look for $\nu_\mu \to \nu_\tau$ with a beam of ν_μ from the KEK accelerator directed at the Super-Kamiokande detector, with more powerful Fermilab-Soudan, KEK-Super-Kamiokande, and possibly CERN-Gran Sasso long-baseline experiments in subsequent years.

Evidence for non-zero neutrino mass evidently favors CHDM, but it also disfavors low-Ω models. Because free streaming of the neutrinos damps small-scale fluctuations, even a little hot dark matter causes reduced fluctuation power on small scales and requires substantial cold dark matter to compensate; thus evidence for even 2 eV of neutrino mass favors large Ω and would be incompatible with a cold dark matter density Ω_c as small as 0.3 [8]. Allowing Ω_ν and the tilt to vary, CHDM can fit observations over a somewhat wider range of values of the Hubble parameter h than standard or tilted CDM [20]. This is especially true if the neutrino mass is shared between two or three neutrino species [8, 21, 22, 23], since then the lower neutrino mass results in a larger free-streaming scale over which the power is lowered compared to CDM; the result is that the cluster abundance predicted with $\Omega_\nu \approx 0.2$ and $h \approx 0.5$ and COBE normalization (corresponding to $\sigma_8 \approx 0.7$) is in reasonable agreement with observations without the need to tilt the model[24] and thereby reduce the small-scale power further. (In CHDM with a given Ω_ν shared between $N_\nu = 2$ or 3 neutrino species, the linear power spectra are identical on large and small scales to the $N_\nu = 1$ case; the only difference is on the cluster scale, where the power is reduced by $\sim 20\%$ [21, 8].)

Another consequence of the reduced power on small scales is that structure formation is more recent in CHDM than in ΛCDM. This may conflict with observations of damped Lyman α systems in quasar spectra, and other observations of protogalaxies at high redshift, although the available evidence does not yet permit a clear decision on this (see below). While the original $\Omega_\nu = 0.3$ CHDM model [25, 26] certainly predicts far less neutral hydrogen in damped Lyman α systems (identified as protogalaxies with circular velocities $V_c \geq 50$ km s^{-1}) than is observed [27, 28], lowering the hot fraction to $\Omega_\nu \approx 0.2$ dramatically improves this [28, 29]. Also, the evidence from preliminary data of a fall-off of

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the amount of neutral hydrogen in damped Lyman α systems for $z \gtrsim 3$ [30] is in accord with predictions of CHDM [28].

However, as for all $\Omega = 1$ models, $h \gtrsim 0.55$ implies $t_0 \lesssim 12$ Gyr, which conflicts with age estimates from globular cluster [31] and white dwarf cooling [32]. The only way to accommodate both large h and large t_0 within the standard FRW framework of General Relativity is to introduce a positive cosmological constant $(\Lambda > 0)$ [33, 34].

ΛCDM flat cosmological models with $\Omega_0 = 1 - \Omega_\Lambda \approx 0.3$, where $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$, were discussed as an alternative to $\Omega = 1$ CDM since the beginning of CDM [1, 35]. They have been advocated more recently [36] both because they can solve the $H_0 - t_0$ problem and because they predict a larger fraction of baryons in galaxy clusters than $\Omega = 1$ models. Early galaxy formation also is often considered to be a desirable feature of these models. But early galaxy formation implies that fluctuations on scales of a few Mpc spent more time in the nonlinear regime, as compared with CHDM models. As has been known for a long time, this results in excessive clustering on small scales. My colleagues and I have found that a typical ΛCDM model with $h = 0.7$ and $\Omega_0 = 0.3$, normalized to COBE on large scales (this fixes $\sigma_8 \approx 1.1$ for this model), is compatible with the number-density of galaxy clusters[24], but predicts a power spectrum of galaxy clustering in real space that is much too high for wavenumbers $k = (0.4 - 1)h/$Mpc [12]. This conclusion holds if we assume either that galaxies trace the dark matter, or just that a region with higher density produces more galaxies than a region with lower density. One can see immediately from Figure 1 that there will be a problem with this ΛCDM model, since the APM power spectrum is approximately equal to the linear power spectrum at wavenumber $k \approx 0.6h$ Mpc^{-1}, so there is no room for the extra power that nonlinear evolution certainly produces on this scale (see Figure 1 of Ref. [12] and further discussion below). The only way to reconcile the model with the observed power spectrum is to assume that some mechanism causes strong anti-biasing — i.e., that regions with high dark matter density produce fewer galaxies than regions with low density. While theoretically possible, this seems very unlikely; biasing rather than anti-biasing is expected, especially on small scales [37]. Numerical hydro+N-body simulations that incorporate effects of UV radiation, star formation, and supernovae explosions [44] do not show any antibias of luminous matter relative to the dark matter.

Our motivation to investigate this particular ΛCDM model was to have H_0 as large as might possibly be allowed in the ΛCDM class of models, which in turn forces Ω_0 to be rather small in order to have $t_0 \gtrsim 13$ Gyr. There is little room to lower the normalization of this ΛCDM model by tilting the primordial power spectrum $P_p(k) = Ak^{n_p}$ (i.e., assuming n_p significantly smaller than the "Zel'dovich" value $n_p = 1$), since then the fit to data on intermediate scales will be unacceptable — e.g., the number density of clusters will be too small [12]. Tilted ΛCDM models with higher Ω_0, and therefore lower H_0 for $t_0 \gtrsim 13$ Gyr, appear to have a better hope of fitting the available data, based on comparing

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quasi-linear calculations to the data [12, 38]. But all cosmological models with a cosmological constant Λ large enough to help significantly with the $H_0 - t_0$ problem are in trouble with new observations providing strong upper limits on Λ [39]: gravitational lensing [40], HST number counts of ellptical galaxies [41], and especially the preliminary results from measurements using high-redshift Type Ia supernovae [42]. The analysis of the data from the first 7 of the Type Ia supernovae from the LBL group [43] gave $\Omega_0 = 1 - \Omega_\Lambda = 0.94^{+0.34}_{-0.28}$, or equivalently $\Omega_\Lambda = 0.06^{+0.28}_{-0.34}$ (< 0.51 at the 95% confidence level).

It is instructive to compare the $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model that we have been considering with standard CDM and with CHDM. At $k = 0.5h$ Mpc^{-1}, Figs. 5 and 6 of Ref. [45] show that the $\Omega_\nu = 0.3$ CHDM spectrum and that of a biased CDM model with the same $\sigma_8 = 0.67$ are both in good agreement with the values indicated for the power spectrum $P(k)$ by the APM and CfA data, while the CDM spectrum with $\sigma_8 = 1$ is higher by about a factor of two. As Figure 2 shows, CHDM with $\Omega_\nu = 0.2$ in two neutrino species [8] also gives nonlinear $P(k)$ consistent with the APM data.

3 Cluster Baryons

I have recently reviewed the astrophysical data bearing on the values of the fundamental cosmological parameters, especially Ω_0 [39]. One of the arguments against $\Omega = 1$ that seemed hardest to answer was the "cluster baryon crisis" [46]: for the Coma cluster the baryon fraction within the Abell radius is

$$f_b \equiv \frac{M_b}{M_{tot}} \geq 0.009 + 0.050h^{-3/2}, \tag{1}$$

where the first term comes from the galaxies and the second from gas. If clusters are a fair sample of both baryons and dark matter, as they are expected to be based on simulations, then this is 2-3 times the amount of baryonic mass expected on the basis of BBN in an $\Omega = 1$, $h \approx 0.5$ universe, though it is just what one would expect in a universe with $\Omega_0 \approx 0.3$. The fair sample hypothesis implies that

$$\Omega_0 = \frac{\Omega_b}{f_b} = 0.33 \left(\frac{\Omega_b}{0.05} \right) \left(\frac{0.15}{f_b} \right). \tag{2}$$

A review of the quantity of X-ray emitting gas in a sample of clusters [47] finds that the baryon mass fraction within about 1 Mpc lies between 10 and 22% (for $h = 0.5$; the limits scale as $h^{-3/2}$), and argues that it is unlikely that (a) the gas could be clumped enough to lead to significant overestimates of the total gas mass — the main escape route considered in [46] (cf. also [48]). If $\Omega = 1$, the alternatives are then either (b) that clusters have more mass than virial estimates based on the cluster galaxy velocities or estimates based on hydrostatic equilibrium [49] of the gas at the measured X-ray temperature
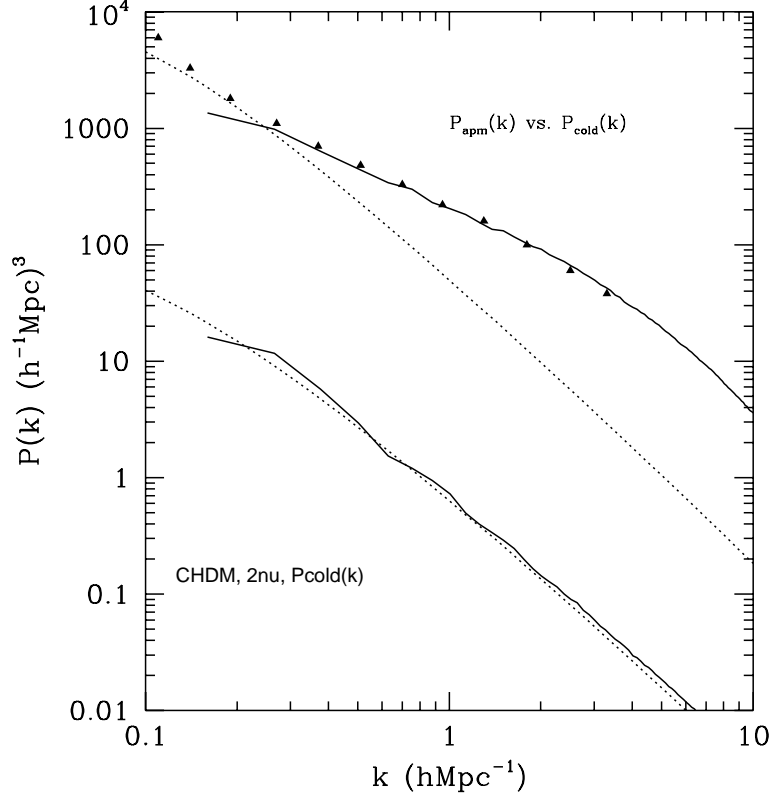
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Figure 2: Comparison of APM galaxy power spectrum (triangles) with nonlinear cold particle power spectrum from CHDM model considered in this paper (upper solid curve). The dotted curves are linear theory; upper curves are for $z = 0$, lower curves correspond to the higher redshift $z = 9.9$. (From Ref. [7].)

(which is surprising since they agree [50]), (c) that the usual BBN estimate $\Omega_b \approx 0.05(0.5/h)^2$ is wrong, or (d) that the fair sample hypothesis is wrong [51]. Regarding (b), it is interesting that there are indications from weak lensing [52] that at least some clusters may actually have extended halos of dark matter — something that is expected to a greater extent if the dark matter is a mixture of cold and hot components, since the hot component clusters less than the cold [53, 54]. If so, the number density of clusters as a function of mass is higher than usually estimated, which has interesting cosmological implications (e.g., σ_8 is a little higher than usually estimated). It is of course possible that the solution is some combination of alternatives (a)-(d). If none of the alternatives is right, then the only conclusion left is that $\Omega_0 \approx 0.33$. The cluster baryon problem is clearly an issue that deserves very careful examination.

It has recently been argued [55] that CHDM models are compatible with

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the X-ray data within observational uncertanties of both the BBN predictions and X-ray data. Indeed, the rather high baryon fraction $\Omega_b \approx 0.1(0.5/h)^2$ implied by recent measurements of low D/H in two high-redshift Lyman limit systems [56, 57] helps resolve the cluster baryon crisis for all $\Omega = 1$ models — it is escape route (c) above. With the higher Ω_b implied by the low D/H, there is now a "baryon cluster crisis" for low-Ω_0 models! Even with a baryon fraction at the high end of observations, $f_b \lesssim 0.2(h/0.5)^{-3/2}$, the fair sample hypothesis with this Ω_b implies $\Omega_0 \gtrsim 0.5(h/0.5)^{-1/2}$.

4   Warm Dark Matter vs. CHDM

It will be instructive to look briefly at Warm Dark Matter (WDM), both to see that some variants of CDM have less success than others in fitting cosmological observations, and also because there is renewed interest in WDM. Although CHDM and WDM are similar in the sense that both are intermediate models between CDM and HDM, CHDM and WDM are quite different in their implications.

The problems with a pure Hot Dark Matter (HDM) adiabatic cosmology are well known: free-streaming of the hot dark matter completely destroys small-scale fluctuations, so that the first structures that can form are on the mass scale of clusters or superclusters, and galaxies must form by fragmentation of these larger structures; but observations show that galaxies are much older than superclusters, which have low overdensity and are still forming. Moreover, with the COBE upper limit to the normalization of HDF, hardly any structure at all will form even by the present epoch.

WDM is a simple modification of HDM, obtained by changing the assumed average number density n of the particles. In the usual HDM, the dark matter particles are neutrinos, each species of which has $n_\nu = 108$ cm^{-3}, with a corresponding mass of $m_0 = \Omega_\nu \rho_0 / n_\nu = \Omega_\nu 92 h^2$ eV, with $\Omega_\nu = 1 - \Omega_b \approx 1$. In WDM, there is a new parameter, m/m_0, the ratio of the mass of the warm particle to the above neutrino mass; correspondingly, the number density of the warm particles is reduced by the inverse of this factor, so that their total contribution to the cosmological density is unchanged. Pagels and I [58] proposed perhaps the first WDM particle candidate, a light gravitino, which was the lightest supersymmetric particle (LSP) in the earliest version of supersymmetric phenomenology (which was subsequently largely abandoned in favor of a hidden-sector sypersymmetry breaking scheme, but which is now being reconsidered: cf. [59]). Olive & Turner [60] proposed left-handed neutrinos as WDM. In both cases, these particles interact much more weakly than neutrinos, decouple earlier from the hot big bang, and thus have diluted number density compared to neutrinos since they do not share in the entropy released by the subsequent annihilation of species such as quarks. This is analogous to (but more extreme than) the neutrinos themselves, which have lower number density

today than photons because the neutrinos decouple before e^+e^- annihilation (and also because they are fermions).

In order to investigate the cosmological implications of any dark matter candidate, it is necessary to work out the gravitational clustering of these particles, first in linear theory, and then after the amplitude of the fluctuations grows into the nonlinear regime. Colombi, Dodelson, & Widrow [61] recently did this for WDM, and Figure 1 in their paper compares the square of the linear transfer functions for WDM and CHDM.

One often can study large scale structure just on the basis of such linear calculations, without the need to do computationally expensive simulations of the non-linear gravitational clustering. Such studies have shown that matching the observed cluster and galaxy correlations on scales of about 20-30 h^{-1} Mpc in CDM-type theories requires that the "Excess Power" $EP \approx 1.3$, where

$$EP \equiv \frac{\sigma(25 \ h^{-1} \ \text{Mpc})/\sigma(8 \ h^{-1} \ \text{Mpc})}{[\sigma(25 \ h^{-1} \ \text{Mpc})/\sigma(8 \ h^{-1} \ \text{Mpc})]_{sCDM}}, \tag{3}$$

and as usual $\sigma(r) = (\delta\rho/\rho)(r)$ is the rms fluctuation amplitude in randomly placed spheres of radius r. The EP parameter was introduced in the COBE-DMR interpretation paper [62], and in a recent paper [24] we have shown that EP is related to the spectrum shape parameter Γ [64] by $\Gamma \approx 0.5(EP)^{-3.3}$. For CHDM and other models, this is a useful generalization since the cluster correlations do seem to be a function of this generalized Γ, with $\Gamma \approx 0.23$ to match cluster correlation data [24]. Peacock & Dodds [63] have shown that $\Gamma \approx 0.23$ also is required to match large scale galaxy clustering data.

Since calculating $\sigma(r)$ is a simple matter of integrating the power spectrum times the top-hat window function,

$$\sigma(r) = \int_0^\infty P(k)W(kr)k^2 dk \tag{4}$$

the linear calculations immediately allow determination of EP for WDM and CHDM. Ref. [61] shows that for WDM to give the required EP, the parameter value $m/m_0 \approx 1.5 - 2$, while for CHDM the required value of the CHDM parameter is $\Omega_\nu \approx 0.3$. But for WDM with $m/m_0 \gtrsim 2$, the spectrum lies a lot lower than the CDM spectrum at $k \gtrsim 0.3h^{-1}$ Mpc (length scales $\lambda \lesssim 20 \ h^{-1}$ Mpc), which in turn implies that formation of galaxies, corresponding to the gravitational collapse of material in a region of size ~ 1 Mpc, will be strongly suppressed compared to CDM. Thus WDM will not be able to accommodate simultaneously the distribution of clusters and galaxies. But CHDM will do much better.

Probably the only way to accommodate WDM in a viable cosmological model is as part of a mixture with hot dark matter, which might even arise naturally in a supersymmetric model [65] of the sort in which the gravitino is the LSP [59]. Cold plus "volatile" dark matter is a related possibility [66].

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There are many more parameters needed to describe the presently available data on the distribution of galaxies and clusters and their formation history than the few parameters needed to specify a CDM-type model. Thus it should not be surprising that at most a few CDM variant theories can fit all this data. Once it began to become clear that standard CDM was likely to have problems accounting for all the data, after the discovery of large-scale flows of galaxies was announced in early 1986 [67], I advised Jon Holtzman in his dissertation research to work out the linear theory for a wide variety of CDM variants [21] so that we could see which ones would best fit the data [22]. The clear winners were CHDM with $\Omega_\nu \approx 0.3$ if $h \approx 0.5$, and ΛCDM with $\Omega_0 \approx 0.2$ if $h \approx 1$. Variants of both these models remain perhaps the best bets still.

5 CHDM: Early Structure Troubles?

Aside from the possibility mentioned at the outset that the Hubble constant is too large and the universe too old for any $\Omega = 1$ model to be viable, the main potential problem for CHDM appears to be forming enough structure at high redshift. Although, as I mentioned above, the prediction of CHDM that the amount of gas in damped Lyman α systems is starting to decrease at high redshift $z \gtrsim 3$ seems to be in accord with the available data, the large velocity spread of the associated metal-line systems *may* indicate that these systems are more massive than CHDM would predict (see e.g., [68, 69]). Also, results from a recent CDM hydrodynamic simulation [70] in which the amount of neutral hydrogen in protogalaxies seemed consistent with that observed in damped Lyman α systems led the authors to speculate that CHDM models would produce less than enough; however, since the regions identified as damped Lyman α systems in the simulations were not actually resolved, this will need to be addressed by higher resolution simulations for all the models considered.

Finally, Steidel et al. [71] have found objects by their emitted light at redshifts $z = 3 - 3.5$ apparently with relatively high velocity dispersions (indicated by the equivalent widths of absorption lines), which they tentatively identify as the progenitors of giant elliptical galaxies. *Assuming* that the indicated velocity dispersions are indeed gravitational velocities, Mo & Fukugita (MF) [72] have argued that the abundance of these objects is higher than expected for the COBE-normalized $\Omega = 1$ CDM-type models that can fit the low-redshift data, including CHDM, but in accord with predictions of the ΛCDM model considered here. (In more detail, the MF analysis disfavors CHDM with $h = 0.5$ and $\Omega_\nu \gtrsim 0.2$ in a single species of neutrinos. They apparently would argue that this model is then in difficulty since it overproduces rich clusters — and if that problem were solved with a little tilt $n_p \approx 0.9$, the resulting decrease in fluctuation power on small scales would not lead to formation of enough early objects. However, if $\Omega_\nu \approx 0.2$ is shared between two species of neutrinos, the resulting model appears to be at least marginally consistent with both clusters

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and the Steidel objects even with the assumptions of MF. The ΛCDM model with $h = 0.7$ consistent with the most restrictive MF assumptions has $\Omega_0 \gtrsim 0.5$, hence $t_0 \lesssim 12$ Gyr. ΛCDM models having tilt and lower h, and therefore more consistent with the small-scale power constraint discussed above, may also be in trouble with the MF analysis.) But in addition to uncertainties about the actual velocity dispersion and physical size of the Steidel et al. objects, the conclusions of the MF analysis can also be significantly weakened if the gravitational velocities of the observed baryons are systematically higher than the gravitational velocities in the surrounding dark matter halos, as is perhaps the case at low redshift for large spiral galaxies [73], and even more so for elliptical galaxies which are largely self-gravitating stellar systems in their central regions.

Given the irregular morphologies of the high-redshift objects seen in the Hubble Deep Field [74] and other deep HST images, it seems more likely that they are relatively low mass objects undergoing starbursts, possibly triggered by mergers, rather than galactic protospheroids. Since the number density of the brightest of such objects may be more a function of the probability and duration of such starbursts rather than the nature of the underlying cosmological model, it may be more useful to use the star formation or metal injection rates [75] indicated by the total observed rest-frame ultraviolet light to constrain models [76]. The available data on the history of star formation [77, 78, 75] suggests that most of the stars and most of the metals observed formed relatively recently, after about redshift $z \sim 1$; and that the total star formation rate at $z \sim 3$ is perhaps a factor of 3 lower than at $z \sim 3$, with yet another factor of ~ 3 falloff to $z \sim 4$ (although the rates at $z \gtrsim 3$ could be higher if most of the star formation is in objects too faint to see). This is in accord with indications from damped Lyman α systems [79] and expectations for $\Omega = 1$ models such as CHDM, but not with the expectations for low-Ω_0 models which have less growth of fluctuations at recent epochs, and therefore must form structure earlier. But this must be investigated using more detailed modelling, including gas cooling and feedback from stars and supernovae [76], before strong conclusions can be drawn.

There is another sort of constraint from observed numbers of high-redshift protogalaxies that would appear to disfavor ΛCDM. The upper limit on the number of $z \gtrsim 4$ objects in the Hubble Deep Field (which presumably correspond to smaller-mass galaxies than most of the Steidel objects) is far lower than the expectations in low-Ω_0 models, especially with a positive cosmological constant, because of the large volume at high redshift in such cosmologies [80]. Thus evidence from high-redshift objects cuts both ways, and it is too early to tell whether high- or low-Ω_0 models will ultimately be favored.

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6 Advantages of Mixed CHDM Over Pure CDM Models

There are three basic reasons why a mixture of cold plus hot dark matter works better than pure CDM without any hot particles: **(1)** the power spectrum shape $P(k)$ is a better fit to observations, **(2)** there are indications from observations for a more weakly clustering component of dark matter, and **(3)** a hot component may help avoid the too-dense central dark matter density in pure CDM dark matter halos. I will discuss each in turn.

(1) Spectrum shape. As I explained in discussing WDM vs. CHDM above, the pure CDM spectrum $P(k)$ does not fall fast enough on the large-k side of its peak in order to fit indications from galaxy and cluster correlations and power spectra. The discussion there of "Excess Power" is a way of quantifying this. This is also related to the overproduction of clusters in pure CDM. The obvious way to prevent $\Omega = 1$ sCDM normalized to COBE from overproducing clusters is to tilt it a lot (the precise amount depending on how much of the COBE fluctuations are attributed to gravity waves, which can be increasingly important as the tilt is increased). But a constraint on CDM-type models that is likely to follow both from the high-z data just discussed and from the preliminary indications on cosmic microwave anisotropies at and beyond the first acoustic peak from the Saskatoon experiment [81] is that viable models cannot have much tilt, since that would reduce too much both their small-scale power and the amount of small-angle CMB anisotropy. As I have already explained, by reducing the fluctuation power on cluster scales and below, COBE-normalized CHDM naturally fits both the CMB data and the cluster abundance without requiring much tilt. The need for tilt is further reduced if a high baryon fraction $\Omega_b \gtrsim 0.1$ is assumed [82], and this also boosts the predicted height of the first acoustic peak. No tilt is necessary for $\Omega_\nu = 0.2$ shared between $N_\nu = 2$ neutrino species with $h = 0.5$ and $\Omega_b = 0.1$. Increasing the Hubble parameter in COBE-normalized models increases the amount of small-scale power, so that if we raise the Hubble parameter to $h = 0.6$ keeping $\Omega_\nu = 0.2$ and $\Omega_b = 0.1(0.5/h)^2 = 0.069$, then fitting the cluster abundance in this $N_\nu = 2$ model requires tilt $1 - n_p \approx 0.1$ with no gravity waves (i.e., $T/S = 0$; alternatively if $T/S = 7(1 - n_p)$ is assumed, about half as much tilt is needed, but the observational consequences are mostly very similar, with a little more small scale power). The fit to the small-angle CMB data is still good, and the predicted Ω_{gas} in damped Lyman α systems is a little higher than for the $h = 0.5$ case. The only obvious problem with $h = 0.6$ applies to any $\Omega = 1$ model — the universe is rather young: $t_0 = 10.8$ Gyr.

(2) Need for a less-clustered component of dark matter. The fact that group and cluster mass estimates on scales of $\sim 1\ h^{-1}$ Mpc typically give values for Ω around 0.1-0.2 [84], while larger-scale estimates give larger values around 0.3-1 [2] suggests that there is a component of dark matter that does

not cluster on small scales as efficiently as cold dark matter is expected to do. In order to quantify this, my colleagues and I have performed the usual group M/L measurement of Ω_0 on small scales in "observed" $\Omega = 1$ simulations of both CDM and CHDM [83]. We found that COBE-normalized $\Omega_\nu = 0.3$ CHDM gives $\Omega_{M/L} = 0.12 - 0.18$ compared to $\Omega_{M/L} = 0.15$ for the CfA1 catalog analyzed exactly the same way, while for CDM $\Omega_{M/L} = 0.34 - 0.37$, with the lower value corresponding to bias $b = 1.5$ and the higher value to $b = 1$ (still below the COBE normalization). Thus local measurements of the density in $\Omega = 1$ simulations can give low values, but it helps to have a hot component to get values as low as observations indicate. We found that there are three reasons why this virial estimate of the mass in groups misses so much of the matter in the simulations: (1) only the mass within the mean harmonic radius r_h is measured by the virial estimate, but the dark matter halos of groups continue their roughly isothermal falloff to at least $2r_h$, increasing the total mass by about a factor of 3 in the CHDM simulations; (2) the velocities of the galaxies are biased by about 70% compared to the dark matter particles, which means that the true mass is higher by about another factor of 2; and (3) the groups typically lie along filaments and are significantly elongated, so the spherical virial estimator misses perhaps 30% of the mass for this reason. Our visualizations of these simulations [53] show clearly how extended the hot dark matter halos are. An analysis of clusters in CHDM found similar effects, and suggested that observations of the velocity distributions of galaxies around clusters might be able to discriminate between pure cold and mixed cold + hot models [54]. This is an area where more work needs to be done — but it will not be easy since it will probably be necessary to include stellar and supernova feedback in identifying galaxies in simulations, and to account properly for foreground and background galaxies in observations.

(3) Preventing too dense centers of dark matter halos. Flores and I [85] pointed out that dark matter density profiles with $\rho(r) \propto r^{-1}$ near the origin from high-resolution dissipationless CDM simulations [87] are in serious conflict with data on dwarf spiral galaxies (cf. also Ref. [86]), and in possible conflict with data on larger spirals [88] and on clusters (cf. [89, 90]). Navarro, Frenk, & White [73] agree that rotation curves of small spiral galaxies such as DDO154 and DDO170 are strongly inconsistent with their universal dark matter profile $\rho_{NFW}(r) \propto 1/[r(r + a)^2]$. I am at present working with Stephane Courteau, Sandra Faber, Ricardo Flores, and others to see whether ρ_{NFW} is consistent with data from high- and low-surface-brightness galaxies with moderate to large circular velocities are consistent with this universal profile. The failure of simulations to form cores as observed in dwarf spiral galaxies either is a clue to a property of dark matter that we don't understand, or is telling us the simulations are inadequate. It is important to discover whether this is a serious problem, and whether inclusion of hot dark matter or of dissipation in the baryonic component of galaxies can resolve it. It is clear that including hot dark matter will decrease the central density of dark matter halos, both because

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the lower fluctuation power on small scales in such models will prevent the early collapse that produces the highest dark matter densities, and also because the hot particles cannot reach high densities because of the phase space constraint [91, 54]. But this may not be enough.

7 Best Bet CDM-Type Models

As I said at the outset, I think CHDM is the best bet if Ω_0 turns out to be near unity and the Hubble parameter is not too large, while ΛCDM is the best bet if the Hubble parameter is too large to permit the universe to be older than its stars with $\Omega = 1$.

Both theories do seem less "natural" than sCDM. But although sCDM won the beauty contest, it doesn't fit the data. CHDM is just sCDM with some light neutrinos. After all, we know that neutrinos exist, and there is experimental evidence — admittedly not yet entirely convincing — that at least some of these neutrinos have mass, possibly in the few-eV range necessary for CHDM.

Isn't it an unnatural coincidence to have three different sorts of matter — cold, hot, and baryonic — with contributions to the cosmological density that are within an order of magnitude of each other? Not necessarily. All of these varieties of matter may have acquired their mass from (super?)symmetry breaking associated with the electroweak phase transition, and when we understand the nature of the physics that determines the masses and charges that are just adjustable parameters in the Standard Model of particle physics, we may also understand why Ω_c, Ω_ν, and Ω_b are so close. In any case, CHDM is certainly not uglier than ΛCDM.

In the ΛCDM class of models, the problem of too much power on small scales that I discussed at some length for $\Omega_0 = 0.3$ and $h = 0.7$ ΛCDM implies either that there must be some physical mechanism that produces strong, scale-dependent anti-biasing of the galaxies with respect to the dark matter, or else that higher Ω_0 and lower h are preferred, with a significant amount of tilt to get the cluster abundance right and avoid too much small-scale power [12]. Higher $\Omega_0 \gtrsim 0.5$ also is more consistent with the evidence summarized above against large Ω_Λ and in favor of larger Ω_0, especially in models such as ΛCDM with Gaussian primordial fluctuations. But then $h \lesssim 0.63$ for $t_0 \gtrsim 13$ Gyr.

Among CHDM models, having $N_\nu = 2$ species share the neutrino mass gives a better fit to COBE, clusters, and small-scall data than $N_\nu = 1$, and moreover it appears to be favored by the available experimental data [8]. But it remains to be seen whether CHDM models can fit the data on structure formation at high redshifts.

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